

SELECTION OF PRODUCTS BASED ON CUSTOMER PREFERENCES APPLYING FUZZY LOGIC

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Abstract: Customer satisfaction depends on many variables, such as quality, price, availability, customer service, and so on, and increases with the degree to which the delivered product meets the customer's preferences. Frequently, vendors have to help customers select a product from among those available to satisfy their needs and wants. Most of the time, the information provided by the customers is not very precise. In this paper, we propose a method to select the product that is the closest to their preferences. A way to measure the relative indifference between different characteristics (Fuzzy Indifference Degree, FID) is proposed as well, which is based on fuzzy preference relations. An example is given to illustrate the applicability of the proposed method.

Key words: product evaluation, product selection, customer satisfaction, fuzzy logic, fuzzy preference relation.

1- Introduction

One of the most important issues for most companies is increasing customer satisfaction. It is strongly related to other variables which are critical for companies to consider, such as the quality, price, and availability of goods and services, customer service, etc. According to Jamali (2005), there is a very strong relationship between quality and the level of customer satisfaction. This author has previously conducted a study about this relationship, in which the quality determinants were grouped into five basic clusters, including reliability, responsiveness, assurance, empathy, and tangibles. A way to calculate customer satisfaction has been proposed by Feciková (2004), which considers four principal elements, such as level of satisfaction, level of importance, type of customer, and type of method used.

At sale/purchase time, customer preferences are regularly expressed in a vague way, and often couched in colloquial or linguistic terms. To select a product for the customer based on this kind of information, various fuzzy techniques can be considered for developing new and more accurate ways to

understand the needs and wants of customers.

2- Literature review

2.1- Fuzzy customer satisfaction

According to Jamali (2005), the concepts of service quality and service satisfaction are closely related, although the exact nature of customer judgments and the relationship between them remains fuzzy. Fortunately, studies have been conducted in which fuzzy logic is applied to measure customer satisfaction. Liu (1995), for example, proposed models to evaluate customer satisfaction using the analytical hierarchy process and fuzzy set theory, and Kuo (1996) proposed a general fuzzy neural network applying a back propagation learning model to measure the level of customer satisfaction.

Different types of membership function have been used in many tools that have been proposed. The triangular membership function is one of the most frequently used, because it is both readily applicable and practicable. Some of the studies that include the triangular membership function in their inference processes are the following:

- two methods by Chen *et al.* (2004) to determine and revise the priority of customer demands, the first classifying customer demands using natural language processing techniques in order to obtain customer expectations, and the second determining the revised priority of the customer demands using a fuzzy logic inference;
- a methodology devised by Kwong *et al.* (2007) to determine the importance of engineering characteristics, as well as their impact in others alike, in which the fuzzy relation and correlation measures between engineering characteristics are

determined based on the fuzzy expert systems approach;

- a method by Foldesi *et al.* (2007) to extend the Kano model for classifying the relationship between customer satisfaction and attribute-level performance and identifying whether or not some of the attributes have a non-linear relationship with satisfaction, using fuzzy numbers to represent the customer assessments;
- a method by Liu *et al.* (2008) to calculate a customer satisfaction index in e-commerce using fuzzy techniques, such as the fuzzy composition operation to evaluate the validity of an e-commerce operation considering the consumer opinion;
- a method by Lai *et al.* (2008) using fuzzy mathematics to rank new customer requirements considering information from competitors.

Other important membership functions, such as the parabola-based function and the Gaussian function have been applied as well. Yuen and Lau (2006) presented a distributed fuzzy qualitative evaluation system using developed fuzzy algorithms to manage complex distributed evaluation scenarios. In this work, the fuzzy normal distribution is characterized by the parabola-based membership function and vertical partition methods. Lin (2007) provided a model of customer satisfaction from a comprehensive perspective in an attempt to use the nonlinear fuzzy neural network model to verify the study assumptions. In that work, the Gaussian membership function is used to infer the membership function from the inputs.

2.2- Fuzzy product evaluation

According to Ozer (2005), there are various factors which influence decision-making in new product evaluation: factors related to the tasks pertaining to task complexity, task importance, information scarcity and task instructions. The decision-makers related factors to consider the expertise and diversity of the involved people. The elicitation-related factors concerning about the way to elicit the opinions for the new product decision makers. And the aggregation-related factors about the way to aggregate different opinions in the new product evaluation. Very little research has been conducted aimed at including fuzzy logic in the product evaluation process. Some of them are the following: the use by Popp and Lodel (1995) of fuzzy multiple criteria analysis and user models in product evaluation where there are no serious interdependencies among the product attributes; the proposal by Liu (1996) of a fuzzy multi-factor and attribute decision-making model to select a product based on information from a customer using fuzzy sets which uses four-level hierarchical structural analysis and a ranking method for the fuzzification and defuzzification processes respectively; and a modified S-curve membership function methodology used by Vasant and Barsoum (2005) to apply fuzzy linear programming to the selection of an optimal unit of products with a higher level of satisfaction.

3- Fuzzy product selection

In this paper, the proposed method for fuzzy product selection is based principally on the analysis of the fuzzy preference relation between the features of products and the customer's preferences. The method used to calculate the preference relation is an adaptation of the method proposed by Tseng and Klein (1989), which was aimed at extending its scope to consider all the possible pairwise situations between two normal fuzzy numbers, such trapezoidal, triangular, or rectangular.

Here, we propose a method to identify the best products for customers based on a set of their preferences, which consists of the following phases.

- 1- Market and technical evaluation of products
- 2- General prioritization of features
- 3- Customer preference consideration
- 4- Product selection procedure

These phases are explained below.

- 1- Market and technical evaluation of products. This evaluation can generally be obtained by the industry concerned from specialized sources. If it is not available, a survey administered by experts can be used as well. Then, this information needs to be represented in fuzzy numbers. This fuzzification process should be performed by those with sufficient knowledge of the industry in question. Figures 2, 3, 4, and 5 show how to represent the features of different products using fuzzy numbers.
- 2- General prioritization of features. A customer survey can be used to obtain a general feature prioritization for the type of product in question. Figure 6 shows how the priority of each feature could be represented using fuzzy numbers. This representation process can be performed considering the ranges of evaluation for each feature given by the customers surveyed.
- 3- Customer preference consideration. Through a few questions phrased in colloquial or linguistic terms, it is possible to obtain the customer preference for each feature. Based on the general prioritization scale, all these preferences should be represented by fuzzy numbers (see Figure 6 and Table 1).
- 4- Product selection procedure. Let $R(A,B)$ be the fuzzy preference relation and $\mu_R(A, B)$ be the membership function representation of $R(A,B)$. According to Tseng and Klein (1989), if the membership degree $\mu_R(A,B)$ is equal to 0.5, then A and B are indifferent. In this paper, we apply their conditional statement in the same way. If the membership degree $\mu_R(A,B)$ is equal to 0.5, then A and B are indifferent, where A and B represent the product feature evaluation and the customer feature

evaluation respectively. That is to say, this pairwise contains the best feature according to customer preferences. To identify the features that are the best and closest to the customer’s preferences, we propose a Fuzzy Indifference Degree (FID) based on the definition of standard deviation, where the best choice corresponds to the smallest FID of the set of products at issue. By considering the population standard deviation definition the FID can be calculated as follows.

$$FID_i = \sqrt{\sum_{j=1}^m [R(A_{ij}, B_{jk}) - (\sum_{j=1}^m R(A_{ij}, B_{jk}) / m)]^2 / m} \quad (1)$$

where

$R(A_{ij}, B_{jk})$ is the fuzzy preference relation between A_{ij} and B_{jk}

$A_{ij} = \{a_{i1}, a_{i2}, \dots, a_{im}\}$ is the set of features (j) for product (i) for all $i=1, 2, \dots, n$, and for all $j=1, 2, \dots, m$.

$B_{jk} = \{b_{1j}, b_{2j}, \dots, b_{mj}\}$ is the set of features (j) for customer (k) for all $j=1, 2, \dots, m$, and for all $k=1, 2, \dots, p$.

In the real world, only in a very few specific cases is it possible to consider the entire population to obtain the standard deviation. In this paper, because of the set of features selected and the number of features considered, the standard deviation can vary depending on the analyst’s judgment, and the FID for a sample can be obtained by adapting equation (1) as follows:

$$FID_i = \sqrt{\sum_{j=1}^m [R(A_{ij}, B_{jk}) - (\sum_{j=1}^m R(A_{ij}, B_{jk}) / m)]^2 / (m-1)} \quad (2)$$

As mentioned previously, if the fuzzy preference relation $R(A,B)$ is equal to 0.5, it means that there is no difference between A and B. That is to say, this pairwise contains the best feature according to customer preference. Then, $R(A,B)=0.5$ is the target, and equation (2) can be modified by substituting the mean of $R(A,B)$ by the target value of $R(A,B)$. This substitution allows the degree of indifference between the product features and the customer preferences to be measured:

$$FID_i = \sqrt{\sum_{j=1}^m [R(A_{ij}, B_{jk}) - 0.5]^2 / (m-1)} \quad (3)$$

Then, the best product (i) for customer (k) is determined by applying equation 4, as follows:

$$BP_k = \min\{FID_1, FID_2, \dots, FID_i\} \quad (4)$$

where BP_k is the best product alternative for customer (k).

Let A and B be two fuzzy numbers which are convex and normal. Then, there exist two notions, the indifference and the dominance between them. If there exists an area of overlap between fuzzy numbers A and B (intersection between A and

B), then the overlap area is defined as the indifference area (see Figure 1). If there exist one or more non-overlap areas between fuzzy numbers A and B, then, for each non-overlap area, either A dominates B or B dominates A (see Figure 1).

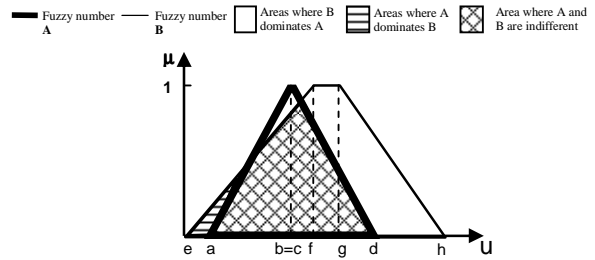


Figure 1: Dominance and indifference between A and B

If A and B are two normal fuzzy numbers, then the fuzzy preference relation $R(A,B)$ or $R(B,A)$ could be obtained using the following equations:

$$R(A, B) = [D(A, B) + I(A, B)] / [A(A) + A(B)] \quad (5)$$

$$R(B, A) = [D(B, A) + I(A, B)] / [A(A) + A(B)] \quad (6)$$

where

$D(A,B)$ is the area where A dominates B,

$D(B,A)$ is the area where B dominates A,

$I(A,B)$ is the area where A and B are indifferent,

$A(A)$ and $A(B)$ are the areas of A and B respectively.

Since $R(A,B)$ and $R(B,A)$ are reciprocal; that is, $R(A, B) + R(B, A) = 1$, then

$$R(B, A) = 1 - R(A, B) \quad (7)$$

$R(A,B)$ and $R(B,A)$ are interpreted as the degree to which A is preferred to B and B is preferred to A respectively.

From Figure 1, the non-overlap areas represent the situations where A dominates B and the areas where B dominates A. In this paper, we use the Hamming distance to calculate these areas.

Let S be an interval in the real line R. Now, the Hamming distance between two fuzzy numbers A and B on S is defined by

$$D(A, B | S) = \int_{u \in S} |\mu_A(u) - \mu_B(u)| du \quad (8)$$

where

$$S = \mathfrak{R}, D(A, B | \mathfrak{R}) = D(A, B) \tag{9}$$

4- Illustrative example

A vendor of laptop computers needs to determine the best product for a customer based on his or her preferences. This vendor has four product alternatives to offer the customer. The vendor’s objective is to define the product that is closest to the customer’s preferences. To achieve this objective, we apply the method proposed in section 3:

1- Market and technical evaluation of products

Let us suppose that a team of specialists has evaluated the product based on the available information about the industry in question. This evaluation was made considering the features depicted in Figure 2, which are denoted F_j , where F_1 denotes speed, F_2 price, F_3 weight, F_4 colour, and F_5 the brand for each laptop alternative.

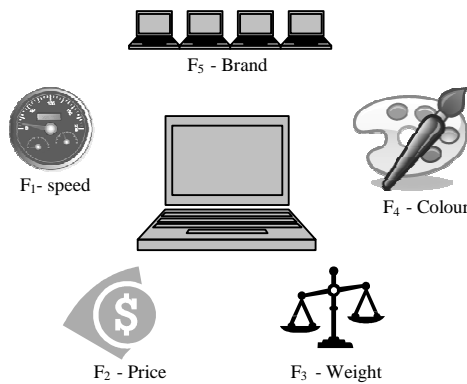


Figure 2: Product features

These features have to be represented by normal fuzzy numbers, as shown in Figures 3, 4, 5, and 6. For all these figures, the features are denoted F_{ij} for all products (i) and for all features (j).

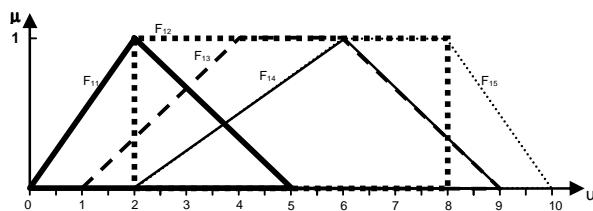


Figure 3: Feature evaluation for product 1

Figure 3 depicts the evaluation of the features for product 1. F_{11} represents the evaluation of the speed feature, given by the specialist team, and is a triangular fuzzy number with the interval (0, 5) as its support. This feature has the value of 3 when its membership function is equal to 1. F_{12} represents the evaluation of the price feature, and is represented by a rectangular fuzzy number with the interval (2, 8) as its support. For this feature, the membership function is equal to 1 for the

entire support interval. F_{13} represents the evaluation of the weight feature, and is a trapezoidal fuzzy number with the interval (1, 9) as its support. It has a membership function equal to 1 for the interval (4, 6). F_{14} represents the evaluation attached to the colour feature. It is a triangular fuzzy number with the interval (2, 9) as its support. It takes the value of 6 when its membership function is equal to 1. The evaluation of the brand feature (F_{15}) is represented by a trapezoidal fuzzy number with the interval (2, 10) as its support. It has a membership function equal to 1 for the interval (6, 8). The feature evaluations in Figures 4, 5, and 6 are represented in the same way.

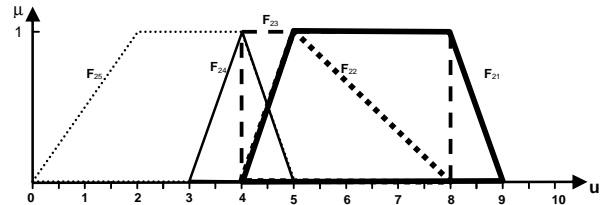


Figure 4: Feature evaluation for product 2

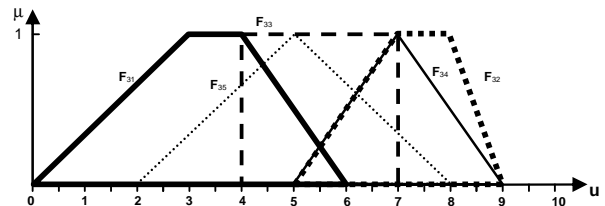


Figure 5: Feature evaluation for product 3

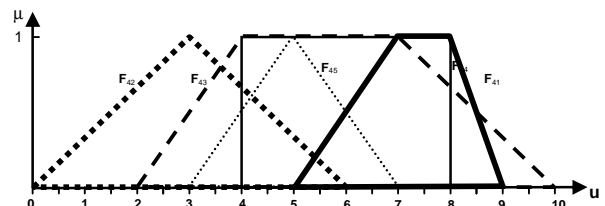


Figure 6: Feature evaluation for product 4

2- General prioritization of features

Let us suppose that a team of specialists defined a general scale based on a customer survey to prioritize the set of considered features, for the type of products in question. This prioritization is depicted in Figure 7.

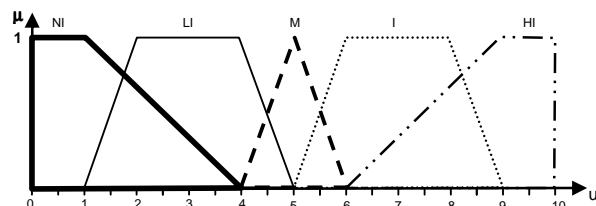


Figure 7: Definition of general feature prioritization

As shown in Figure 7, five different levels are defined in colloquial or linguistic terms:

- HI denotes ‘highly important’ and its fuzzy representation is [6 9 10 10],
- I denotes ‘important’ and its fuzzy representation is [5 6 8 9],
- M denotes ‘a medium level of preference’ and its fuzzy representation is [4 5 5 6],
- LI denotes ‘of low importance’ and its fuzzy representation is [1 2 4 5],
- NI denotes ‘not important’ and its fuzzy representation is [0 0 1 4].

3- Customer preference consideration

Suppose that a vendor has to satisfy the preferences of three different customers to the greatest extent possible. Table 1 presents the preferences of each customer for each feature. This information is expressed in linguistic terms.

Table 1: Customer feature preferences

Feature (F _j)	Feature preferences for customer (C _k)		
	C ₁	C ₂	C ₃
F ₁ - speed	HI	I	HI
F ₂ - price	I	HI	NI
F ₃ - weight	M	I	I
F ₄ - colour	LI	M	HI
F ₅ - brand	NI	LI	I

Table 1 shows the feature preferences for each customer. For customer 1, speed is a highly important feature, price is important, weight is a feature with a medium level of preference, colour is of low importance, and brand is not important. For customer 2, price is a highly important aspect, weight and speed are two important features, colour is an aspect with a medium level of preference, and brand is an aspect of low importance. For customer 3, speed and colour are two highly important features, weight and brand are important, and price is not important.

4- Product selection procedure

First, it is necessary to obtain the fuzzy preference relation between product features and customer preferences. These preferences can be calculated by applying equation (5). Appendices 1, 2, and 3 present the fuzzy preferences relation for each customer.

Second, let us obtain the FID for each product and customer. As presented in Tables 2, 3, and 4, this figure can be obtained using equations 2 and 3.

Table 2: Fuzzy indifference degree per product for customer 1

Product (i)	FID Eq. (2)	FID Eq. (3)
1	0.4004	0.4011
2	0.2907	0.2913
3	0.3974	0.4229
4	0.4298	0.4333

From Table 2, the FID from equation 2 represents the degree of dispersion about the fuzzy preference mean. Even if the fuzzy preference values oscillate between 0 and 1, it does not mean that its value is 0.5. For this, it would be necessary to apply equation 3, since it is the target value that will identify the closest alternative to the customer preferences. The same interpretation applies to Tables 3 and 4.

Table 3: Fuzzy indifference degree per product for customer 2

Product (i)	FID Eq. (2)	FID Eq. (3)
1	0.3806	0.4066
2	0.1862	0.2937
3	0.4092	0.4135
4	0.3365	0.3365

Table 4: Fuzzy indifference degree per product for customer 3

Product (i)	FID Eq. (2)	FID Eq. (3)
1	0.3597	0.3984
2	0.4151	0.4683
3	0.3877	0.4331
4	0.2663	0.3376

Finally, by applying equation (4), it is possible to identify the best product for each customer.

Table 5: Product selection for each customer

Customer (k)	Best product (i) alternative
1	2
2	2
3	4

Table 5 shows that product 2 is the best alternative for customers 1 and 2, and product 4 is the best one for customer 3.

5- Conclusions

A method for identifying the best product alternative for a specific customer is proposed in this paper, aimed at contributing to the customer’s satisfaction increment. This method consists of various phases, such as product evaluation, feature prioritization, customer preference consideration, and product selection. For the product selection procedure, a Fuzzy Indifference Degree was proposed as well, to identify the best choice alternative for a

specific customer. As shown in section 4, the proposed method makes it possible to select the best alternative by considering vague information from customers. That is to say, by applying fuzzy logic techniques, it is possible to make better and more accurate decisions according to customer preferences.

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Appendix 1: Features fuzzy preference relation per customer 1

$F_{ij} \setminus C_{ik}$	C_{11} [6 9 10 10]	C_{21} [5 6 8 9]	C_{31} [4 5 5 6]	C_{41} [1 2 4 5]	C_{51} [0 0 1 4]
F_{11} [0 2 2 5]	0.0000				
F_{12} [2 2 8 8]		0.2778			
F_{13} [1 4 6 9]			0.5000		
F_{14} [2 6 6 9]				0.8615	
F_{15} [2 6 8 10]					0.9619
F_{21} [4 5 8 9]	0.1731				
F_{22} [4 5 5 8]		0.2250			
F_{23} [4 4 8 8]			0.6999		
F_{24} [3 4 4 5]				0.7499	
F_{25} [0 2 4 5]					0.7333
F_{31} [0 3 4 6]	0.0000				
F_{32} [5 7 8 10]		0.5833			
F_{33} [4 4 7 7]			0.6250		
F_{34} [5 7 7 9]				0.9999	
F_{35} [2 5 5 8]					0.9394
F_{41} [5 7 8 9]	0.2250				
F_{42} [0 3 3 6]		0.0000			
F_{43} [2 4 7 10]			0.6154		
F_{44} [4 4 8 8]				0.9286	
F_{45} [3 5 5 7]					0.9778

Appendix 3: Features fuzzy preference relation per customer 3

$F_{ij} \setminus C_{ik}$	C_{13} [6 9 10 10]	C_{23} [0 0 1 4]	C_{33} [5 6 8 9]	C_{43} [6 9 10 10]	C_{53} [5 6 8 9]
F_{11} [0 2 2 5]	0.0000				
F_{12} [2 2 8 8]		0.9216			
F_{13} [1 4 6 9]			0.2500		
F_{14} [2 6 6 9]				0.1250	
F_{15} [2 6 8 10]					0.4375
F_{21} [4 5 8 9]	0.1731				
F_{22} [4 5 5 8]		1.0000			
F_{23} [4 4 8 8]			0.3571		
F_{24} [3 4 4 5]				0.0000	
F_{25} [0 2 4 5]					0.0000
F_{31} [0 3 4 6]	0.0000				
F_{32} [5 7 8 10]		1.0000			
F_{33} [4 4 7 7]			0.2500		
F_{34} [5 7 7 9]				0.2000	
F_{35} [2 5 5 8]					0.1875
F_{41} [5 7 8 9]	0.2250				
F_{42} [0 3 3 6]		0.7576			
F_{43} [2 4 7 10]			0.3529		
F_{44} [4 4 8 8]				0.1026	
F_{45} [3 5 5 7]					0.1333

Appendix 2: Features fuzzy preference relation per customer 2

$F_{ij} \setminus C_{ik}$	C_{12} [5 6 8 9]	C_{22} [6 9 10 10]	C_{32} [5 6 8 9]	C_{42} [4 5 5 6]	C_{52} [1 2 4 5]
F_{11} [0 2 2 5]	0.0000				
F_{12} [2 2 8 8]		0.0784			
F_{13} [1 4 6 9]			0.2500		
F_{14} [2 6 6 9]				0.6444	
F_{15} [2 6 8 10]					0.8875
F_{21} [4 5 8 10]	0.4667				
F_{22} [4 5 5 8]		0.0741			
F_{23} [4 4 8 8]			0.3571		
F_{24} [3 4 4 5]				0.1250	
F_{25} [0 2 4 5]					0.4615
F_{31} [0 3 4 6]	0.0000				
F_{32} [5 7 8 9]		0.2250			
F_{33} [4 4 7 7]			0.2500		
F_{34} [5 7 7 9]				0.9453	
F_{35} [2 5 5 8]					0.8125
F_{41} [5 7 8 10]	0.5833				
F_{42} [0 3 3 6]		0.0000			
F_{43} [2 4 7 10]			0.3529		
F_{44} [4 4 8 8]				0.6999	
F_{45} [3 5 5 7]					0.8666